Smoothness Morrey spaces and their envelopes

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Abstract

The classical Morrey space $\mathcal{M}_{u,p}(\mathbb{R}^n)$, $0 < p \leq u < \infty$, is defined to be the set of all locally p-integrable functions f such that

$$\|f \mid \mathcal{M}_{u,p}(\mathbb{R}^n)\| := \sup_{x \in \mathbb{R}^n, R>0} R^{\frac{n}{u}-\frac{n}{p}} \left(\int_{B(x,R)} |f(y)|^p \, \mathrm{d}y \right)^{\frac{1}{p}}$$

is finite, where $B(x,R)$ denotes the ball centered at $x \in \mathbb{R}^n$ with radius $R > 0$. They are part of the wider class of Morrey-Campanato spaces and can be considered as an extension of the scale of L_p spaces. Built upon these basic spaces Besov-Morrey spaces $\mathcal{N}^s_{u,p,q}$ and Triebel-Lizorkin-Morrey spaces $\mathcal{E}^s_{u,p,q}$ attracted some attention in the last years, in particular in connection with Navier-Stokes equations. Closely related to theses scales are the spaces of Besov type $B^{s,\tau}_{p,q}$ and Triebel-Lizorkin type $F^{s,\tau}_{p,q}$, $\tau \geq 0$, which coincide with their classical counterparts when $\tau = 0$.

We present a survey on such different scales of smoothness spaces of Morrey type. We also introduce the general concept of growth and continuity envelope of a function space and determine the envelopes of the above mentioned spaces. In some cases a specific behaviour appears which is different from the "classical" situation in Besov or Triebel-Lizorkin spaces.

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